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Deconstructing noncommutativity with a giant fuzzy moose

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ABSTRACT: We argue that the worldvolume theories of D-branes probing orbifolds with discrete torsion develop, in the large quiver limit, new non-commutative directions. This provides an explicit 'deconstruction' of a wide class of noncommutative theories. This also provides insight into the physical meaning of discrete torsion and its relation to the T-dual B field. We demonstrate that the strict large quiver limit reproduces the matrix theory construction of higher-dimensional D-branes, and argue that finite 'fuzzy moose' theories provide novel regularizations of non-commutative theories and explicit string theory realizations of gauge theories on fuzzy tori. We also comment briefly on the relation to $NCOS$, $(2,0)$ and little string theories.

KEYWORDS: Lattice Gauge Field Theories, M(atrix) Theories, Brane Dynamics in Gauge Theories, Non-Commutative Geometry.

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1. Introduction

Recent work on the phenomenology of large-Ngauge theories has revealed that theories based on 'moose diagrams' (aka 'quiver theories') generate extra dimensions in the large quiver limit. Explicitly, the lagrangian takes the form of a lattice field theory plus extra irrelevant matter that completes the theory in the UV. This so-called 'dimensional deconstruction' [1] (see also [2, 5]) has an elegant realization in string theory, where the quiver theories arise as the worldvolume theories of D-branes on geometric orbifolds.[1]

Some rather clever moose phonomenology has led to novel presentations of several interesting systems, including the mysterious interacting $6d$ $(2,0)$ and little string theories [6], hinting that there is more to be learned from this subtle regulation scheme.

Of course, there already exist methods for generating higher-dimensional branes, in particular matrix theory constructions [7, 8, 9, 13] which lead to non-commutative worldvolumes. What is the relation between these two approaches? Can one deconstruct non-commutative dimensions?

[1] Earlier work along related lines includes the beautiful work of [3] which argued that the strong coupling of the T-dual of IIB on A_N should be read, in the large-N limit, as the decompaction of the M-Theory circle; retracing this argument on the coulomb branch of little strings on tori, Ganor and Sethi observed that particular strong-coupling limits of associated matrix theories should reproduce higher-dimensional SYM at strong coupling and large-N. While this is different from "deconstruction", it is very similar physics and teaches several very similar lessons. See also [4], which studied $\mathrm{U}(k)^N$ theories with bifundamental matter and found a light vector in the large-N limit but missed the lovely physical interpretation.

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In this note we present evidence that the worldvolume theories of D-branes probing $\mathbb{C}^3/\mathbb{Z}_N \times \mathbb{Z}_N$ orbifolds with discrete torsion provide, in the $N \to \infty$ large-moose limit, precisely such a construction. The new non-commutative directions appear exactly as in the original case; their non-commutativity derives directly from the extra phases specifying the discrete torsion.

This can be motivated as follows. First, as we will explain in detail below, the partition function for closed strings on the torus $\mathbb{R}^2/\mathbb{Z} \times \mathbb{Z}$ with discrete torsion $\epsilon = e^{2\pi i/n}$, $1/n \in (0,1)$, is identical to that on the same torus but with a constant background B-field such that $\epsilon = e^{2\pi i \int B}$. Thus the theory of a D0-brane on a torus with discrete torsion ϵ is dual to the theory of a D2-brane wrapping the T-dual torus with a (rescaled) constant background B-field $b \sim 1/n$ — which is exactly noncommutative SYM on the same torus.

Now consider the orbifold $\mathbb{C}^3/\mathbb{Z}_N \times \mathbb{Z}_N$ with discrete torsion ϵ. This is a cone over base $S^5/\mathbb{Z}_N \times \mathbb{Z}_N$; for large N this is a very sharp cone. Consider a thin region of this cone far from the orbifold fixed point, where the geometry is approximately $\mathbb{R}^4 \times T^2$; in the $N \to \infty$ limit, the local physics precisely reproduces the toroidal orbifold with discrete torsion. This implies that the quiver theory of a Dp-brane on $\mathbb{C}^3/\mathbb{Z}_N \times \mathbb{Z}_N$ with discrete torsion reproduces, in the large-Nlimit, the noncommutative theory of a D$(p+2)$-brane wrapping a torus with a constant background B-field specified by the discrete torsion.

In the following we explicitly verify this conjecture by demonstrating that this quiver theory is equivalent to SYM on an N^2-point fuzzy torus with a noncommutativity parameter specified by the discrete torsion $\Theta \sim 1/b \sim n$.[2] (In this relationship we keep the closed string volume fixed.) In the $N \to \infty$ limit, this precisely reproduces SYM on a smooth torus with constant B-field correctly specified by the discrete torsion. In the language of [1], this shows that the orbifold $\mathbb{C}^3/\mathbb{Z}_N \times \mathbb{Z}_N$ with discrete torsion can be used to "deconstruct" a wide class of noncommutative theories.

In particular, one can use the $\mathbb{C}^3/\mathbb{Z}_N \times \mathbb{Z}_N$ orbifolds with discrete torsion to deconstruct noncommutative Dp-brane theories for $p = 2, 3, 4, 5$. For $p = 3$, one can take a further strong coupling limit to obtain a deconstruction of NCOS theory [10]. For $p = 4, 5$, the D-brane SYM theories are not UV complete; it is a remarkable fact that the deconstructed theories appear to contain precisely the degrees of freedom required to complete the D-brane theories to $(2,0)$ [11] or little string theories [12], respectively, as demonstrated in [6].

We begin by reviewing the original 'deconstruction' phenomenon [6]. We then recall the quiver theories on branes probing orbifolds with discrete torsion and demonstrate how they deconstruct non-commutative theories. We compare the deconstruction of noncommutative D-branes to their matrix theory constructions, finding agreement in the large moose limit, and argue that finite moose provide stringy realizations of gauge theories on fuzzy tori. We close with further speculations and open questions. (For related earlier work, see e.g. [24, 25, 26].)

[2]Everywhere in the paper we choose to describe NCYM in terms of parameters given by eqations (3.30), (3.31).

2. A brief review of (de)construction

Consider the worldvolume theory of a single D0-brane on a supersymmetric $\mathbb{C}^2/\mathbb{Z}_N$ orbifold. (A D3-brane probe of this orbifold was used in [6] to deconstruct the six-dimensional (2,0) theory.) The orbifold can be thought of as a local model for an A_{N-1} singularity in a K3 manifold, preserving half of the original supersymmetry. A general technique for constructing worldvolume theories of D-branes on orbifolds was described in a remarkable paper by Douglas and Moore [14]; we follow their procedure.

Parameterizing the target space with five real scalars x^m, $m = 1, \ldots, 5$ and two complex scalars $z^1 = x^6 + ix^7$ and $z^2 = x^8 + ix^9$, the geometric action of the $\mathbb{Z}_N$ generator is

$$R(e) = \exp\left(\frac{2\pi i(J_{67} - J_{89})}{N}\right). \tag{2.1}$$

The massless worldvolume fields of the parent $\mathcal{N} = 4$ U(N) gauge theory are gauge fields A^0_{ij}, scalars X^m_{ij}, Z^1_{ij}, Z^2_{ij}, and Majorana-Weyl spinors in the **16** of SO(9,1), λ_{ij}. As in [15], we construct χ, a Weyl spinor of SO(5,1), out of the components of λ having $(s_{67}, s_{89}) = (-\frac{1}{2}, -\frac{1}{2})$ or $(s_{67}, s_{89}) = (+\frac{1}{2}, +\frac{1}{2})$. Similarly the Weyl spinor η will contain the components of λ with either $(s_{67}, s_{89}) = (-\frac{1}{2}, +\frac{1}{2})$ or $(s_{67}, s_{89}) = (\frac{1}{2}, -\frac{1}{2})$.

When acting on D-branes, the orbifold group may have an additional action on the chan-paton indices. The action of the generator e of the $\mathbb{Z}_N$ orbifold group can thus be written

$$|\psi, i, j\rangle \to \gamma(e)_{ii'}\ |R(e)\psi, i', j'\rangle\ \gamma(e)^{-1}_{j'j}\,, \tag{2.2}$$

where the γ-matrices belong to a faithful representation of the orbifold group. In the case at hand, we can express $\gamma(e)$ in a convenient basis as

$$\gamma(e) = \mathrm{diag}(0, e^{2\pi i/N}, \ldots, e^{2\pi i(N-1)/N})\,. \tag{2.3}$$

The fields surviving the orbifold projection (2.2) are thus

$$A^0_{ii}\,, \qquad X^m_{ii}\,, \qquad Z^1_{i,i+1}\,, \qquad Z^2_{i+1,i}\,, \qquad \chi_{i,i}\,, \qquad \eta_{i,i-1}\,. \tag{2.4}$$

This spectrum can be conveniently represented by so called moose or quiver diagrams (see figure 1).

The classical potential, descending from the potential of the parent theory by restricting to fields which survive the orbifold projection, includes the following term for every $m = 1, \ldots, 5$:

$$V = \frac{1}{2gNl_s(2\pi l_s^2)^2}\sum_{j=0}^{N}\left(X^m_{j+1,j+1} - X^m_{jj}\right)^2\left(|Z^1_{j,j+1}|^2 + |Z^2_{j+1,j}|^2\right). \tag{2.5}$$

There are also terms quartic in Z^1 and Z^2 forcing $|Z^1_{j,j+1}|$ and $|Z^2_{j+1,j}|$ to be independent of j. The moduli space has a coulomb branch, where all $Z^1_{j,j+1}$ and $Z^2_{j+1,j}$ vanish and where X^m_{jj} can be independently varied. It also has a higgs branch with

$$|Z^1_{j,j+1}| = r_1\,, \qquad |Z^2_{j,j+1}| = r_2\,. \tag{2.6}$$

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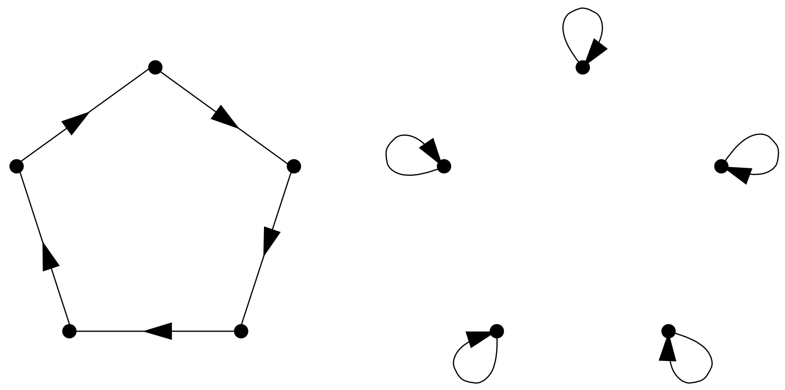

Figure 1: The $\mathbb{C}^2/\mathbb{Z}_5$ Moose. On the left is the moose for $Z^1_{i,i+1}$, $\bar{Z}^2_{i,i+1}$, and $\bar{\eta}_{i,i+1}$. On the right is the moose for A^0_{ii}, X^m_{ii}, and $\chi_{i,i}$.

We have added a factor of $1/|\mathbb{Z}_N| = 1/N$ to (2.5) so that the higgs expectation values (2.6) correspond to moving a D0-brane (consisting of N fractional D0-branes) to a distance $r = (r_1^2 + r_2^2)^{-1/2}$ from the orbifold point.

The trick is now to study low energy fluctuations around a particular point on the higgs branch given by some fixed values of r_1 and r_2. The leading order potential for X^m_{jj}, for example, will be

$$V = \frac{r^2}{2gNl_s(2\pi l_s^2)^2}\sum_{j=0}^{N}\left(X^m_{j+1,j+1} - X^m_{jj}\right)^2 . \tag{2.7}$$

For large N and X_{jj} slowly varying with j, this looks very much like a lattice discretization of

$$V = \frac{2\pi R r^2}{gN^2 l_s(2\pi l_s^2)^2}\int_0^{2\pi R}\frac{1}{2}\left(\frac{dX^m}{d\sigma}\right)^2 d\sigma \tag{2.8}$$

with $\sigma \in (0, 2\pi R)$ and effective lattice spacing $a = 2\pi R/N$. It is a stimulating and life-affirming exercise to check that the rest of the lagrangian takes the correct form to reproduce, in the continuum limit, the worldvolume theory of a D1-brane.

There is a simple geometric reason that this works. For large N the orbifold is a sharp cone over base $S^3/\mathbb{Z}_N \sim S^2 \times (S^1/\mathbb{Z}_N)$. For large r, far from the fixed locus, the local geometry seen by the D0-brane is approximately a cylinder of radius $r_c = r/N$, with the compact coordinate being in the $J_{67} - J_{89}$ direction [6, 15]. T-duality along this coordinate (which is valid far from the fixed locus) produces a D1-brane wrapping an S^1 of radius

$$R \equiv \tilde{r}_c = \frac{l_s^2}{r_c} = \frac{Nl_s^2}{r} \tag{2.9}$$

and string coupling

$$\tilde{g}_s = \frac{R}{l_s}g_s = \frac{l_s}{r_c}g_s = \frac{Nl_s}{r}g_s . \tag{2.10}$$

With these values of R and $\tilde{g}_s$, the factor in front of the integral in (2.8) becomes precisely the D1-brane tension

$$\tau_1 = \frac{1}{\tilde{g}_s l_s (2\pi l_s)} \,. \tag{2.11}$$

It is simple and remarkable to check that the interaction terms in the D1-brane worldvolume theory thus obtained are correctly normalized.

We can of course T-dualize without approximating the geometry by a cylinder [16]. The T-dual of the full orbifold geometry involves N NS5-branes evenly spaced along the T-dual circle, while the fractional D0-branes at the orbifold point become D-strings stretched between the NS5-branes.

2.1 Scalings

It is easy to find similar quiver theories which mock up higher dimensional D-branes at energies lower than $1/a$ (alternatively, whose IR physics in the limit of $a \to 0$ reproduces a higher-dimensional D-brane worldvolume theory). In supersymmetric cases, the number of supercharges is increased in this limit, because an orbifold generally breaks some supersymmetry, whereas a torus with periodic boundary conditions does not. In our example, the number of supercharges is doubled, giving 16 real supercharges in the end.

The scalings of the various couplings also deserve attention. Consider for concreteness the deconstruction of $5d$ U(k) theory with small but finite coupling g_5^2. This proceeds as above by studying light fluctuations off the higgs branch of the cyclic moose, similar to [1]. Let g_q be the gauge coupling in the original U$(k)^N$ quiver theory. Out along the higgs branch, the effective coupling of the surviving U(k) gauge theory is

$$g_4^2 = \frac{g_q^2}{N} \,. \tag{2.12}$$

The relation between the $5d$ and $4d$ couplings is as usual for kaluza-klein reduction,

$$g_5^2 = g_4^2 R \,. \tag{2.13}$$

Expressed in terms of the coupling in the original quiver theory, the $5d$ coupling is thus

$$g_5^2 = \frac{g_q^2 R}{N} \,, \tag{2.14}$$

so holding the $5d$ coupling and (emergent) radius R fixed and finite while taking $N \to \infty$ gives finite g_4 but requires taking the original coupling of the higgsed moose large,

$$g_q \sim N \,. \tag{2.15}$$

The upshot is that, while the gauge coupling of the original quiver theory is getting large, the gauge couplings of both the k Dp-branes far form the orbifold fixed point and of the deconstructed D$(p+2)$-branes can be kept arbitrarily weak.

More generally, consider the deconstruction of a $(d+p)$ dimensional theory with small but finite 't hooft coupling kg_{p+d}^2. In terms of the effective lattice spacing a and quiver coupling g_q,

$$g_{p+d}^2 \approx a^d g_q^2 \,. \tag{2.16}$$

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The (very strongly higgsed!) original quiver theory is thus strongly coupled in the continuum limit $a \to 0$. On the other hand, for N large but fixed energy scale comparable to $R \sim r/N$ in the deconstructed theory, these configurations have extremely large higgs expectation values $r \sim N$, with the result that the low-energy excitations are not localized in the moose but spread over many gauge groups. This makes the effective interactions small, so the dynamics can be studied perturbatively, which is the statement that the effective $(p+d)$-dimensional 't hooft coupling kg^2_{p+d} can be held weak in the continuum limit.

Second, the matrix hamiltonian, which gives for example (2.5), is usually said to be valid only for small separations between the D-branes. Naively, one might worry that (2.5) becomes inapplicable when r approaches the string length l_s. However, as was discussed in e.g. [9], the true limitation is to energies lower than the string scale, i.e. to strings shorter than l_s. This requires

$$\frac{r}{N} \ll l_s \tag{2.17}$$

so that the model will be rich enough to accurately describe the physics. This bound (2.17) translates to requiring the lattice spacing a to be *larger* than the string length,

$$a \gg l_s\,. \tag{2.18}$$

Happily, this is not an obstacle, as we are interested in the decoupling limit, i.e. in the physics at energies much lower than $1/l_s$.

2.2 $\mathbb{C}^3/\mathbb{Z}_N \times \mathbb{Z}_N$ without discrete torsion

Finally, let's review some of the salient properties of the $\mathbb{C}^3/\mathbb{Z}_N \times \mathbb{Z}_N$ orbifold *without* discrete torsion. (This orbifold was used to deconstruct (1,1) little string theory in [6].) Choosing three real transverse coordinates x^1, x^2, x^3 and three complex coordinates $z^1 = x^4 + ix^5$, $z^2 = x^6 + ix^7$, $z^3 = x^8 + ix^9$ the geometric action of the two orbifold group generators is

$$\begin{aligned} R(e_1) &= \exp(2\pi i(-J_{67} + J_{89})/N) \\ R(e_2) &= \exp(2\pi i(J_{45} - J_{89})/N)\,, \end{aligned} \tag{2.19}$$

preserving one quarter of the original supersymmetry. The field content corresponding to k transverse type II Dp-branes ($p \leq 3$) descends from a configuration of kN^2 Dp-branes in the parent theory; a judicious choice of basis gives the action on gauge indices as

$$\begin{aligned} \gamma(e_1)_{aj\tilde{k};\,a'j'\tilde{k}'} &= (e^{2\pi i/N})^j\ \delta_{aa'}\delta_{jj'}\delta_{\tilde{k}\tilde{k}'} \\ \gamma(e_2)_{aj\tilde{k};\,aj'\tilde{k}'} &= (e^{2\pi i/N})^{\tilde{k}}\ \delta_{aa'}\delta_{jj'}\delta_{\tilde{k}\tilde{k}'}\,, \end{aligned} \tag{2.20}$$

where $a, a' = 1, \ldots, k$ and $j, j', \tilde{k}, \tilde{k}' = 1, \ldots, N$. The surviving spectrum is summarized by the quiver diagram on figure 2.

In $4d$ $\mathcal{N} = 1$ language, $3N^2$ bifundamental chiral superfields survive, plus adjoints from vector multiplets for each of the N^2 U(k) gauge groups. (Of course, if $p < 3$, we have to dimensionally reduce these fields.)

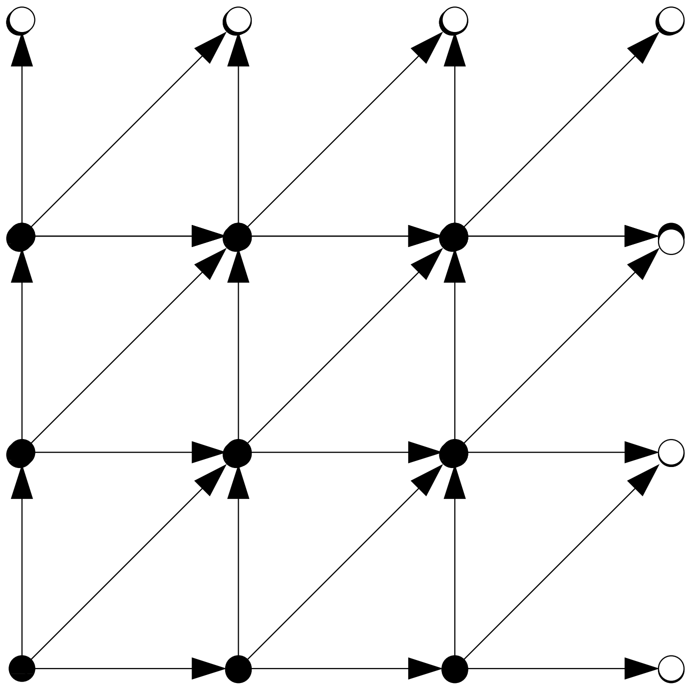

Figure 2: $\mathbb{C}^3/\mathbb{Z}_N \times \mathbb{Z}_N$ Moose for Bifundamental Scalars. Note that it is periodic.

Giving identical vevs to all 'horizontal' and 'vertical' bifundamentals in the quiver diagram generates two discretized dimensions forming a rectangular torus. Giving a further vev to the 'diagonal' bifundamentals gives instead a slanted torus. In both cases, the resulting low energy theory can be thought of as a latticized version of the worldvolume theory of k wrapped D$(p+2)$-branes. (See [6] for details).

3. Discrete torsion and fuzzy moose

As we have seen, Dp-branes on $\mathbb{Z}_N \times \mathbb{Z}_N$ orbifolds deconstruct D$(p+2)$-branes. We will argue that discrete torsion makes the two new world-volume coordinates non-commutative in an appropriate continuum limit. We begin with a review of quiver theories on D-branes probing orbifolds with discrete torsion, discuss the basic strategy, and proceed with an explicit example.

First let us review the physical meaning of discrete torsion, relate it to a T-dual B-field, and give an overview of the logical structure of our construction.

3.1 Discrete torsion as a T-dual B-field

Orbifolds with discrete torsion [19] (see [22] for a beautiful mathematical presentation) generalize geometric orbifolds by adding to the twisted sectors of the path integral phases which depend on the orbifold group elements defining that sector. Modular invariance forces the discrete torsion to lie in $H^2(\Gamma, \mathrm{U}(1))$, i.e. the torsion is a two-cocycle of the orbifold group.

A trivial example is the torus, $T^2 = \mathbb{R}^2/\mathbb{Z}\times\mathbb{Z}$, whose partition function is a sum over winding $\equiv$ twisted sectors

$$\Box = \sum_{(a,b\,|\,a',b')} \Box_{(a,b\,|\,a',b')}\,. \tag{3.1}$$

Adding discrete torsion ammounts to adding phases of the form

$$\Box = \sum_{(a,b\,|\,a',b')} \Box_{(a,b\,|\,a',b')}\, e^{2\pi i(a\cdot b'-b\cdot a')/n}\,. \tag{3.2}$$

Importantly, this is identical to the partition function for the torus with a constant background longitudinal B-field with $b = 1/n \in (0,1)$, so the torus with discrete torsion is identical to the torus with constant background B-field. This fits with the fact that the B-field takes values in $H^2(T^2, \mathrm{U}(1))$.

Let's consider the $\mathbb{C}^3/\mathbb{Z}_N\times\mathbb{Z}_N$ orbifold with discrete torsion $\epsilon \in H^2(\mathbb{Z}_N\times\mathbb{Z}_N, \mathrm{U}(1)) = \mathbb{Z}_N$ labeled by the integer m' (mod N) in $\epsilon = e^{2\pi i m'/N} \equiv e^{2\pi i/n}$. For N large, this is a sharp cone over base $S^5/\mathbb{Z}_N\times\mathbb{Z}_N$, which in particular contains a T^2 factor. By taking $N\to\infty$ (and $\lim_{N\to\infty} m'/N$ fixed) while moving away from the orbifold fixed point so as to keep the volume of the torus fixed, we recover the background $\mathbb{R}^4\times T^2$, where the twisted sectors of the orbifold become the winding sectors on the torus, so the discrete torsion of the orbifold becomes discrete torsion on the torus.

Now probe the $\mathbb{C}^3/\,\mathbb{Z}_N\times\mathbb{Z}_N$ torsion orbifold with a D0-brane far from the fixed point and again consider the constant volume, large-N limit. From the above, this limit is identical to the theory of a D0-brane on the same torus in the presence of a background B-field. T-dualizing both legs of the torus gives a D2-brane wrapping the dual torus with a rescaled background B-field, whose worldvolume theory is SYM on a noncommutative torus with noncommutativity given by $1/b$. Thus we can realize noncommutative SYM as the large-N limit of the quiver theory of a D-brane on a $\mathbb{C}^3/\,\mathbb{Z}_N\times\mathbb{Z}_N$ orbifold with discrete torsion.

Detailed study of this quiver theory reveals that, in this limit, the theory becomes precisely SYM on an N^2-point fuzzy torus, with noncommutativity given in terms of the volume of the torus and the discrete torsion. But the large-N limit of this fuzzy SYM is exactly SYM on a noncommutative torus. Thus the quiver theory of a Dp-brane probing a $\mathbb{C}^3/\,\mathbb{Z}_N\times\mathbb{Z}_N$ with discrete torsion precisely deconstructs the worldvolume theory of a D$(p+2)$-brane wrapping a noncommutative torus. This can be expressed in the commuting diagram (figure 3).

This provides a simple and useful physical interpretation of discrete torsion.[3] Since the possibility of including discrete torsion depends on the non-vanishing of $H_2(\Gamma, \mathrm{U}(1))$, having discrete torsion means that the orbifold can be presented as a fibration whose fibres contain 2-cycles. The interpretation suggested by our analysis is that the discrete torsion should be understood as the B-field along (the T-dual of) each fibre. The utility of this interpretation is that it applies just as well in the case of orbifolds with fixed points (at which points the fibration becomes singular), where fractional branes can wrap the

[3]We would partiuclarly like to thank E. Silverstein and K Dasgupta for discussions on these points.

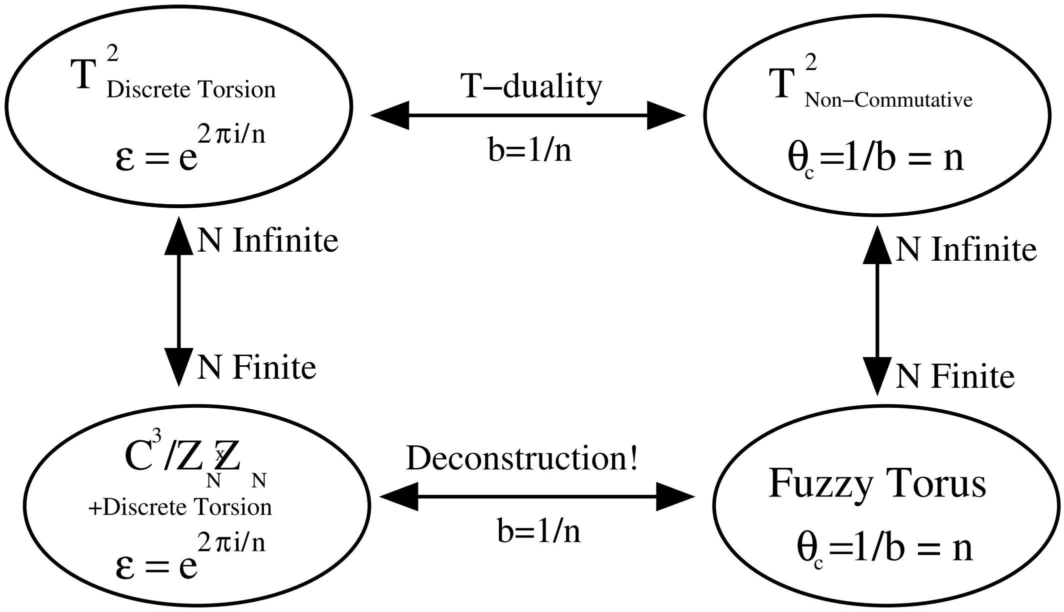

Figure 3: Deconstruction is T-Duality. Note that $n \equiv m'/N$. The scalings indicated are for fixed volume of the torus seen by closed strings.

shrunken cycle, as to orbifolds with freely acting orbifold groups (non-singular fibrations), such as $T^2 \equiv \mathbb{R}^2 / \mathbb{Z} \times \mathbb{Z}$, where there is no shrunken cycle to wrap.

In the remainder of this section we explicitly verify the above story for $\mathbb{C}^3 / \mathbb{Z}_N \times \mathbb{Z}_N$.

3.2 What moose know about discrete torsion

As explained in [20, 21, 18], chan-paton indices in the worldvolume theories of D-branes probing orbifolds with discrete torsion transform in projective representations of the orbifold group, $\gamma(g)\gamma(g') = \tilde{\epsilon}(g, g')\gamma(gg')$, where the phase $\tilde{\epsilon}$ again lies in $H^2(\Gamma, \mathrm{U}(1))$. In the following we focus on the orbifold group $\Gamma = \mathbb{Z}_N \times \mathbb{Z}_N$, for which $H^2(\mathbb{Z}_N \times \mathbb{Z}_N, \mathrm{U}(1)) \cong \mathbb{Z}_N$, so the choice of discrete torsion is specified by one number, m. For N and m relatively prime (which we will assume throughout for simplicity), there is a unique irreducible projective representation of the orbifold group, which can be realized as $N \times N$ matrices involving the usual clock and shift operators. To get a representation describing k Dp-branes on the orbifold, we tensor this irreducible representation with $kN \times kN$ matrices.

We emphasize that this phenomenon works for general B-fields. To motivate this, consider deconstruction of a general manifold $\mathcal{M}$ via an orbifold with quantum symmetry Γ such that $\Gamma \to \mathcal{M}$ in the continuum limit. The discrete torsion is classified by $H^2(\Gamma, \mathrm{U}(1))$; in the continuum limit this becomes $H^2(\mathcal{M}, \mathrm{U}(1))$, which classifies the B-field along the emergent dimensions.

3.3 Fuzzy D-branes from $\mathbb{C}^3 / \mathbb{Z}_N \times \mathbb{Z}_N$ orbifolds with discrete torsion

The physics of D-branes in the supersymmetric $\mathbb{C}^3 / \mathbb{Z}_N \times \mathbb{Z}_N$ orbifold with discrete torsion was described in [21]. The two-cocycle classes $\tilde{\epsilon}^{\,m}$ of $H^2(\Gamma, \mathrm{U}(1)) \cong \mathbb{Z}_N$ will be represented by

$$(\mathbb{Z}_N \times \mathbb{Z}_N) \times (\mathbb{Z}_N \times \mathbb{Z}_N) \to \mathrm{U}(1)$$
$$((a,b),(a',b')) \to \zeta^{m(ab'-a'b)}\,, \tag{3.3}$$

where $\zeta = e^{\pi i/N}$ for N even and $\zeta = e^{2\pi i/N}$ for N odd, $m \in \{0, 1, \ldots, N-1\}$ labels the possible choices of discrete torsion, and we restrict for simplicity to N and m relatively prime (as in [21]). With these conventions, $\epsilon \equiv \zeta^{2m}$ generates $\mathbb{Z}_N$. The geometric action of the orbifold group is

$$R(e_1) = \epsilon^{-J_{67}+J_{89}}\,, \qquad R(e_2) = \epsilon^{J_{45}-J_{89}}\,. \tag{3.4}$$

We will be interested in the case of k D($p \leq 3$)-branes, consisting of kN^2 fractional Dp-branes, probing the orbifold. In the parent U(kN^2) SYM theory, we have, in the $\mathcal{N} = 1, d = 4$ language, one vector multiplet $\hat{A}$ and three chiral superfields $\hat{\Phi}_1, \hat{\Phi}_2$ and $\hat{\Phi}_3$. (In general, we will use a hat to denote matrices $kN^2 \times kN^2$.)

The $\hat{\gamma}$-matrices acting on the chan-paton sector can be chosen as

$$\hat{\gamma}(e_1) = \mathbb{1}_{k\times k} \otimes \mathbb{1}_{N\times N} \otimes U, \qquad \hat{\gamma}(e_2) = \mathbb{1}_{k\times k} \otimes \mathbb{1}_{N\times N} \otimes V\,. \tag{3.5}$$

We define shift and clock matrices U and V, satisfying $UV = \epsilon VU$, as in [21] (where they were called P and Q). For odd N,

$$U = \begin{pmatrix} 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & 0 & 1 \\ 1 & 0 & 0 & \ldots & 0 \end{pmatrix} \qquad V = \begin{pmatrix} 0 & \epsilon & 0 & \ldots & 0 \\ 0 & 0 & \epsilon^2 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & 0 & \epsilon^{n-1} \\ 1 & 0 & 0 & \ldots & 0 \end{pmatrix}. \tag{3.6}$$

For even N, U is as above and V is defined using $\delta^2 = \epsilon$ as

$$V = \begin{pmatrix} 0 & \delta & 0 & \ldots & 0 \\ 0 & 0 & \delta^3 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & 0 & \delta^{2n-3} \\ \delta^{2n-1} & 0 & 0 & \ldots & 0 \end{pmatrix}. \tag{3.7}$$

The fields left invariant by the full orbifold action are of the form

$$\hat{A} = A \otimes \mathbb{1}\,, \qquad \hat{\Phi}_1 = \Phi_1 \otimes U\,, \qquad \hat{\Phi}_2 = \Phi_2 \otimes V\,, \qquad \hat{\Phi}_3 = \Phi_3 \otimes (VU)^{-1}\,. \tag{3.8}$$

The superpotential of the parent theory may be written

$$W = \frac{\tau_p(g_s)}{4(2\pi l_s^2)^2} \hat{\mathrm{Tr}} \left(\hat{\Phi}_1 \hat{\Phi}_2 \hat{\Phi}_3 - \hat{\Phi}_1 \hat{\Phi}_3 \hat{\Phi}_2 \right), \tag{3.9}$$

where $\tau_p(g_s)$ is the Dp-brane tension for string coupling g_s. The orbifold-projected superpotential is then

$$W = \frac{\tau_p(g_s)}{4N(2\pi l_s^2)^2} \mathrm{Tr} \left(\Phi_1 \Phi_2 \Phi_3 - \epsilon^{-1} \Phi_1 \Phi_3 \Phi_2 \right), \tag{3.10}$$

where we have added a factor of $1/|\Gamma|$ as in section 2. The F- and D-terms are

$$V_F = \frac{\tau_p(g_s)}{4N(2\pi l_s^2)^2}\left(|\Phi_1\Phi_2-\epsilon^{-1}\Phi_2\Phi_1|^2+|\Phi_2\Phi_3-\epsilon^{-1}\Phi_3\Phi_2|^2+|\Phi_3\Phi_1-\epsilon^{-1}\Phi_1\Phi_3|^2\right), \quad (3.11)$$

$$V_D = \frac{\tau_p(g_s)}{16N(2\pi l_s^2)^2}\,\mathrm{Tr}\left([\Phi_1,\Phi_1^\dagger]^2+[\Phi_2,\Phi_2^\dagger]^2+[\Phi_3,\Phi_3^\dagger]^2\right), \quad (3.12)$$

where $|M|^2$ means $\mathrm{Tr}(MM^\dagger)$. In the continuum limit,

$$\Phi_1 = z_1\mathbb{1}_{k\times k}\otimes V\,, \qquad \Phi_2 = z_2\mathbb{1}_{k\times k}\otimes U\,, \qquad \Phi_3 = z_3\mathbb{1}_{k\times k}\otimes (VU)^{-1}\,, \quad (3.13)$$

corresponds to a stack of coincident D$(p+2)$-branes; for simplicity we fix $z_3=0$.

$$\Phi_1 = z_1\mathbb{1}_{k\times k}\otimes V\,, \qquad \Phi_2 = z_2\mathbb{1}_{k\times k}\otimes U\,, \qquad \Phi_3 = 0\,. \quad (3.14)$$

Note that (3.13) has zero energy and thus lies on the moduli space.

It is the spectrum of light fluctuations off the moduli space that reveals the presence of emergent dimensions. Focussing for clarity on scalars, we rewrite the bosonic parts Z_1 and Z_2 of the chiral superfields Φ_1 and Φ_2 as

$$Z_1 = (r_1\mathbb{1}_{kN\times kN}+H_1)\,e^{i\omega_1}L_1\tilde{V}\,, \qquad Z_2 = (r_2\mathbb{1}_{kN\times kN}+H_2)\,e^{i\omega_2}L_2\tilde{U}\,, \quad (3.15)$$

where H_1, H_2 are hermitian $kN\times kN$ matrices, L_1, L_2 are $kN\times kN$ unitary matrices, and

$$z_1 = r_1e^{i\omega_1}\,, \qquad z_2 = r_2e^{i\omega_2}\,, \qquad \tilde{U} = \mathbb{1}_{k\times k}\otimes U\,, \qquad \tilde{V} = \mathbb{1}_{k\times k}\otimes V\,. \quad (3.16)$$

With these conventions, the background (3.14) corresponds to $H_1=H_2=0$ and $L_1=L_2=\mathbb{1}_{kN\times kN}$.

Substituting these into (3.11), we get, among other terms,

$$-\frac{\tau_p(g_s)r_1^2r_2^2}{2N(2\pi l_s^2)^2}\epsilon^{-1}\,\mathrm{Tr}\left(L_2\tilde{U}\,L_1\tilde{V}\,\tilde{U}^\dagger L_2^\dagger\,\tilde{V}^\dagger L_1^\dagger-\mathbb{1}_{kN\times kN}\right)+\text{c.c.} \quad (3.17)$$

This is precisely the plaquette operator of U(k) gauge theory on a fuzzy torus [23], with L_1 and L_2 being the usual link variables! It is a remarkable fact that the full set of fluctuations flesh out a certain fuzzy torus gauge theory: besides the U-V part (3.17) of "$F_{\mu\nu}F^{\mu\nu}$," we can identify the p-U and p-V parts coming form the kinetic terms for Φ_1 and Φ_2. Together with the original p-p piece, this forms the kinetic term of a gauge field living in p continuous spacetime dimensions and two discrete dimensions forming a fuzzy torus.

Further, H_1 and H_2 appear as adjoint scalars on this fuzzy torus, the extra pieces of their kinetic terms appearing in (3.11) and (3.12). For example, the D-term gives "$(\mathcal{D}_VH_1)^2$", while "$(\mathcal{D}_UH_1)^2$" originates from the F-term.

There are also terms involving other fields and other interactions which one might not have expected in a simple gauge theory on a fuzzy torus. While some of them reflect the fact that changing different expectation values effectively deforms the fuzzy torus (e.g. from a flat to a slanted torus), some do not have an immediately obvious interpretation. This should probably not be too much of a surprise, as the most naive stringy realization of gauge theory on a fuzzy torus, i.e. the matrix theory construction, is unstable.

3.4 Scalings of the fuzzy moose

The procedure for taking the continuum limit of fuzzy geometries is standard and will not be repeated here. Our interest now lies in finding the scalings of various physical quantities in this limit. To identify the lattice spacing, we will use the fact that the normalization of the term in the lagrangian involving $\dot{L}_1$ and $\dot{L}_2$ is

$$\frac{\tau_p(g_s)}{2N}\,\mathrm{Tr}\left(r_1^2(\dot{L}_1\tilde{V})(\dot{L}_1\tilde{V})^\dagger + r_2^2(\dot{L}_2\tilde{U})(\dot{L}_2\tilde{U})^\dagger\right). \tag{3.18}$$

Comparing (3.17) and (3.18) to the normalization in [23] we get

$$a^{\tilde{2}} = \frac{2\pi l_s^2}{r_2}, \qquad a^{\tilde{1}} = \frac{2\pi l_s^2}{r_1}. \tag{3.19}$$

The radii and volume of the torus are

$$R^{\tilde{2}} = \frac{Na^{\tilde{2}}}{2\pi} = \frac{Nl_s^2}{r_2}, \qquad R^{\tilde{1}} = \frac{Na^{\tilde{1}}}{2\pi} = \frac{Nl_s^2}{r_1}, \tag{3.20}$$

$$V_o = N^2 a^{\tilde{1}} a^{\tilde{2}} = \frac{(2\pi l_s^2)^2 N^2}{r_1 r_2}. \tag{3.21}$$

The most relevant quantity, of course, is the emergent noncommutativity parameter $\theta^{\tilde{1}\tilde{2}}$. For this purpose, we formally write

$$U = e^{ix^{\tilde{2}}/R^{\tilde{2}}}, \qquad V = e^{ix^{\tilde{1}}/R^{\tilde{1}}}, \tag{3.22}$$

$$[x^{\tilde{1}}, x^{\tilde{2}}] = i\theta^{\tilde{1}\tilde{2}}. \tag{3.23}$$

Since we defined $\epsilon \equiv \zeta^{2m}$, and $\zeta = e^{\pi i/N}$ for N even and $\zeta = e^{2\pi i/N}$ for N odd, the relation $UV = \epsilon VU$ implies

$$\theta^{\tilde{1}\tilde{2}} = \frac{2\pi m}{N} R^{\tilde{1}} R^{\tilde{2}} \quad (N \text{ odd}), \qquad \theta^{\tilde{1}\tilde{2}} = \frac{4\pi m}{N} R^{\tilde{1}} R^{\tilde{2}} \quad (N \text{ even}). \tag{3.24}$$

From the overall normalization and using the results of [23], we can also read off the gauge coupling

$$G^2_{ym,p+2} = \frac{1}{\tau_p(g_s) l_s^4} R^{\tilde{1}} R^{\tilde{2}}. \tag{3.25}$$

We will continue this discussion in section 3.5 after relating the fuzzy moose to the matrix theory construction of higher-dimensional D-branes.

3.5 Large-N matrix theory vs. the giant fuzzy moose

The standard matrix theory construction of k noncompact D$(p+2)$-branes with $N \to \infty$ Dp-branes [7, 8] begins with the ordinary matrix lagrangian in $\mathbb{R}^{10}$ and expands around a background of $kN \times kN$ matrices satisfying

$$[x^{\tilde{1}}, x^{\tilde{2}}] = i\theta^{\tilde{1}\tilde{2}}, \tag{3.26}$$

the other $x^{\tilde{i}}$ being zero.[4] The matrices $x^{\tilde{1}}$ and $x^{\tilde{2}}$ should generate the space of N^2 linearly independent matrices, so that any $kN \times kN$ matrix can be expressed as a $k \times k$ matrix whose entries are functions of $x^{\tilde{1}}$ and $x^{\tilde{2}}$. The background (3.26) satisfies the equations of motion, and by studying its fluctuations, one recovers the non-commutative field theory describing the higher-dimensional D-branes.

Note that the matrix potential can be obtained by dimensional reduction of the kinetic term and the superpotential of $\mathcal{N} = 4, d = 4$ SYM theory. Expressed in the $\mathcal{N} = 1, d = 4$ variables, the superpotential is

$$W = \frac{\tau_p(g_{\rm mat})}{4(2\pi l_s^2)^2} \,\mathrm{Tr}(\Phi_1\Phi_2\Phi_3 - \Phi_1\Phi_3\Phi_2)\,, \tag{3.27}$$

which reproduces (3.10) in the limit $\epsilon \to 1$, if we choose $g_{\rm mat} = N g_s$.

A finite N analog of (3.26) can be constructed with the same matrices as in sections 3.3 and 3.4, i.e.

$$\Phi_1 = z_1 \mathbb{1}_{k\times k} \otimes V\,, \qquad \Phi_2 = z_2 \mathbb{1}_{k\times k} \otimes U\,, \qquad \Phi_3 = 0\,. \tag{3.28}$$

However, this background does not minimize the potential and will evolve with time. For this reason, we should talk only about very large D-branes, for which the decay is slow. Alternatively, one might add other terms to the potential, which would stabilize (3.28). To our knowledge, such a stable construction has not been realized within string theory.

Let's compare this to the construction in sections 3.3 and 3.4. The backgrounds about which we expand, (3.14) and (3.28), are formally identical, differing in the ϵ^{-1} factor in the orbifold superpotential (3.10). In general, this is an important distinction. On the other hand, we are free to take $\epsilon \to 1$ ($m/N \to 0$). In this limit, the physics of the two approaches should be the same. Indeed, this can be explicitly checked. In both cases we end up with a stack of D$(p+2)$-branes. Since we want to keep $\theta^{\tilde{1}\tilde{2}}$ in (3.24) fixed, the D$(p+2)$-branes will be very large, decompactifying in the strict limit. This is precisely the situation in which the matrix theory configuration becomes stable.

3.6 Fuzzy math and Morita equivalence

Now, we would like to compare the scalings of various parameters in the matrix theory (for very large $D(p+2)$-branes) to the quiver theory scalings found in section 3.4. At first sight, they seem manifestly different; the matrix background (3.28) describes a torus with radii $r_1 = |z_1|$ and $r_2 = |z_2|$, while the fuzzy moose radii (3.20) are inversely proportional to r_1 and r_2!

Sober second thoughts reveal that it is incorrect to compare the radii in this way. The radii r_1 and r_2 are as measured by the closed string metric, while those in (3.20) should be compared to open string quantities. More precisely, recall that there is an infinite number of possible descriptions of non-commutative theories [27, 8], differing by the choice of the two-form $\Phi'_{\tilde{i}\tilde{j}}$ (not to be confused with the chiral superfields Φ) appearing in the commutation relations

$$[x^{\tilde{i}}, x^{\tilde{j}}] = i\theta^{\tilde{i}\tilde{j}}\,, \quad [\partial_{\tilde{i}}, x^{\tilde{j}}] = i\delta_{\tilde{i}}^{\tilde{j}}\,, \quad [\partial_{\tilde{i}}, \partial_{\tilde{j}}] = -i\Phi'_{\tilde{i}\tilde{j}} \qquad (\tilde{i}, \tilde{j} = \tilde{1}, \tilde{2})\,. \tag{3.29}$$

[4]For simplicity we consider generating two noncommutative dimensions.

As explained in [8], the choice which matrix theory naturally selects is

$$\Phi' = -B\,. \tag{3.30}$$

(This applies also to the fuzzy moose theory.) For this value of Φ', the relation between open and closed string parameters is

$$\theta = B^{-1}\,, \qquad G = -(2\pi l_s^2)^2 B g^{-1} B\,, \qquad G_s = g_s \det(2\pi l_s^2 B g^{-1})^{\frac{1}{2}}\,. \tag{3.31}$$

G and G_s are the open string metric and coupling, respectively, while g and g_s denote their closed string counterparts. Here we condense notation, manipulating matrices as if they had only indices $\tilde{1}$ and $\tilde{2}$ and suppressing other components. Using the continuum results of [8] in a frame where

$$g_{\tilde{i}\tilde{j}} = \eta_{\tilde{i}\tilde{j}}\,, \qquad x^{\tilde{1}} \in (0, 2\pi r_1)\,, \quad x^{\tilde{2}} \in (0, 2\pi r_2) \tag{3.32}$$

we can express the B-field along the brane as [8]

$$B_{\tilde{1}\tilde{2}} = \frac{2\pi N}{V_c} = \frac{N}{2\pi r_1 r_2}\,, \tag{3.33}$$

where

$$V_c = (2\pi)^2 r_1 r_2 \tag{3.34}$$

is the volume of the torus as seen by closed strings. The open string metric (from 3.31) is

$$G_{\tilde{i}\tilde{j}} = (2\pi l_s^2)^2 \frac{N^2}{(2\pi r_1 r_2)^2} \eta_{\tilde{i}\tilde{j}} \tag{3.35}$$

giving the volume seen by open strings

$$V_o = \frac{(2\pi l_s^2)^2 N^2}{r_1 r_2}\,. \tag{3.36}$$

The corresponding gauge coupling is

$$G_{ym,p+2} = \frac{1}{\tau_p(g_s)} \frac{N^2}{r_1 r_2}\,, \tag{3.37}$$

where we have used $g_{\mathrm{mat}} = N g_s$, as identified in the previous section.

These are exactly the results given by the fuzzy moose, provided we set $m = 1$ in choosing the discrete torsion. At first sight this is somewhat disconcerting; why does the moose have this extra parameter that does not appear in the strictly infinite-N matrix theory, and what does it mean physically, anyway? Are the fuzzy moose with different m really different theories?

The resolution comes from morita equivalence in the non-commutative theory, which derives from T-duality in the original theory. As promulgated by Seiberg and Witten [27], morita equivalence relates a noncommutative theory on a flat torus with metric G, gauge coupling g_{ym} and rational theta parameter $\Theta = m/N$ to a commutative theory with parameters

$$\Theta' = 0\,, \qquad G' = \frac{G}{N^2}\,, \qquad g'_{ym} = g_{ym} N^{1/2}\,. \tag{3.38}$$

As the rescalings of the metric and gauge coupling do not depend on m, the fuzzy moose with the different m we consider are all morita equivalent to the same commutative theory on the same torus with the same coupling, and thus equivalent to each other.

This equivalence must be read with a bit of care. For finite N, T-duality on the orbifold is more subtle than on the cylinder (in particular, since winding is conserved only mod N, the dual momenum is conserved only mod N), so the morita equivalence may be only approximate. This in fact seems necessary, since the orbifold theories with discrete torsion for any finite N and different m (again all relatively prime) appear manifestly different — the surviving $\mathbb{Z}_N$ quantum symmetry groups are embedded differently in the $\mathbb{Z}_N \times \mathbb{Z}_N$, and the quiver theory superpotentials contain different phases. It is only in the strict large-Nlimit that this naive T-duality is exact and the theories become truly identical. Happily, it is precisely in this limit that we compare to the strictly infinite-N matrix theory construction [8], which has no such parameter in the first place. This remarkable agreement provides further evidence that the fuzzy moose agrees with matrix theory in the large-Nlimit. That they disagree slightly at finite N again should not be worrying: for finite N, the matrix theory background does not solve the F-term constraints and is thus not stable, while the moose theory is. Physically, the two presentations are essentially two different regularizations of the noncommutative theory which need agree only in the deregulated limit — which they do.

4. Conclusion and open problems

We have presented considerable evidence that quiver theories living on D-branes probing orbifolds with discrete torsion deconstruct higher-dimensional non-commutative theories in the large-moose limit. The lagrangian of the moose theory far along its higgs branch reproduces, for large moose, the lagrangian (3.11), (3.17) for the fuzzy torus. In the strict large moose limit this becomes a gauge theory on a noncomutative torus. The fuzzy moose agrees with the matrix theory construction in the strict large-Nlimit.

It is remarkable that the fuzzy moose is completely well defined for all N, providing a novel and consistent regularization of noncommutative theories and an explicit realization in string theory of gauge theory on a fuzzy torus. This begs the question of how the fuzzy moose encodes the UV/IR correspondence of the continuum non-commutative theory. For example, non-SUSY fuzzy moose theories should deconstruct non-SUSY noncommutative theories, in which we expect IR poles in physical processes arising from UV degrees of freedom [28]; how are these divergences regulated in the fuzzy moose theory? This should be a fruitful ground for exploration.

One immediate extension of our construction is to note that since the strong coupling limit of $4d$ $\mathcal{N} = 4$ SYM is NCOS theory, taking the strong coupling limit of the fuzzy moose D1-branes should provide an explicit deconstruction of NCOS theory. It is also tempting to speculate about theories one might (de)construct from more baroque orbifold geometries. Along these lines, it seems that similar arguments might be used in orbifolds of the conifold with discrete torsion, which have been studied extensively in [24].

Additionally, while the fuzzy moose on D2- or D3-branes naively deconstructs non-commutative $D4$- and $D5$-brane theories, arguments similar to those in [6] suggest that they should actually deconstruct some UV completion of these theories, namely some generalization of $(2,0)$ and little string theories, or perhaps even some more general 5-brane theory with a 3-form generalization of non-commutativity.

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